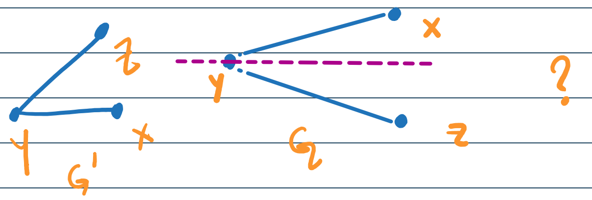
Automorphism : Like the story of 3 friends.

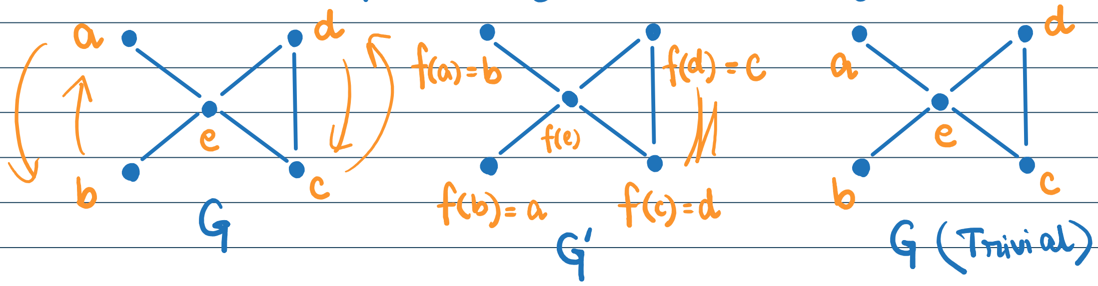
(x, y, z) x, z had a falling out
At a party, you can only recognize y (friendly with both)



Both possible. Automorphisms!
(Notice the symmetry)

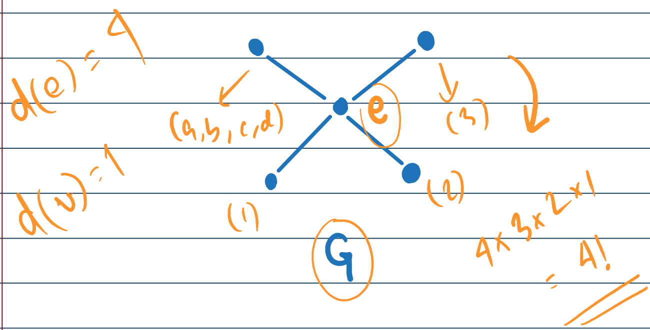
Mathematically, an automorphism of G is a bijection
$f : V(G) \longrightarrow V(G)$ such that $uv \in E(G)$ iff $f(u)f(v) \in E$

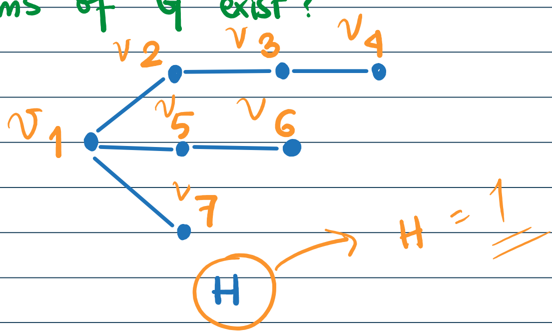
" Measure of how symmetrical the graph is "



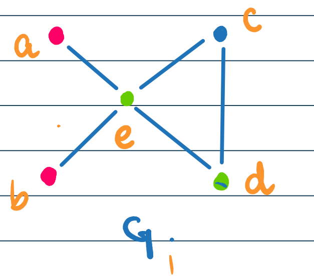
Proof

Q: How many automorphisms of G exist?





Goal : We will try to break the symmetries.



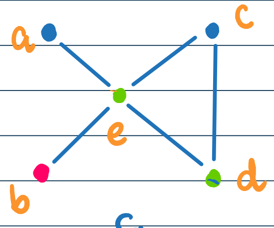
Vertex Coloring : understandable

Asymmetric VC : The only automorphism that preserves the colour classes is the identity automorphism.

Q : Is this vertex coloring asymmetric?

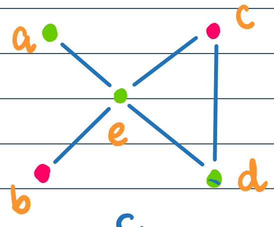
Q.1

2, (1)



G.

Q: what is the min no. of colours required for an asymmetric VC of G ?
= asymm. VC no.



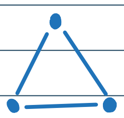
G.

As the colorings cannot be less than 2 (we saw non-trivial automorphisms exist) hence Asymm. VC no. = 2.
= a(G)

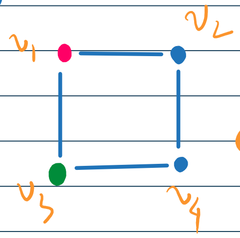
Application : Understanding the symmetries of a graph and computing the assym. colourings is a key step in algorithmic study and pattern matching problems.

Examples (computation) →

1> Cyclic graphs (C_n) :



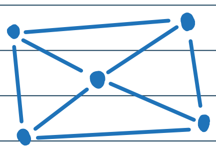
C_3



C_4

(3)

2> Complete graphs (K_n) :



K_4

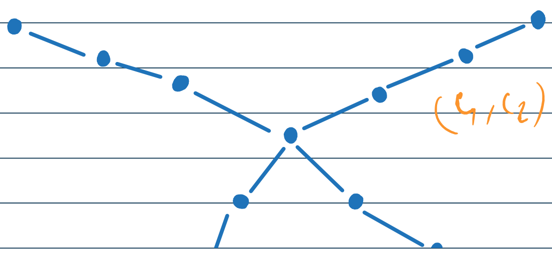
(4)

a(G # H)

a(H)

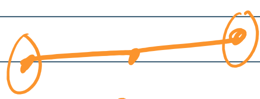
a(G)

3> (Try!) Generalised star ($S_{n,k}$):

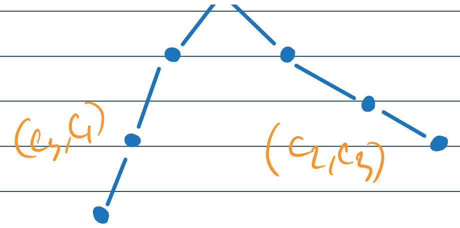


(c_1, c_2)

$S_{4,3}$



P_n



Possible qs: $\rightarrow$

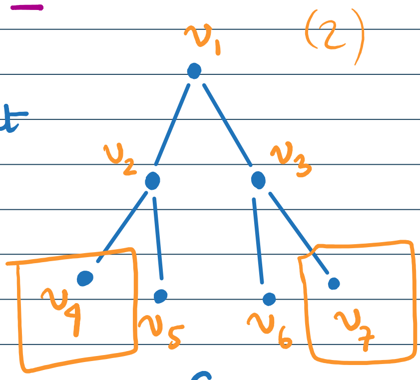
1> Can a graph (under specific conditions) can be broken down into components whose $a(G_i)$'s can be used to find $a(G)$?

2> What about Graph products? or Disjoint Unions?

There exists other methods to break symmetry:—

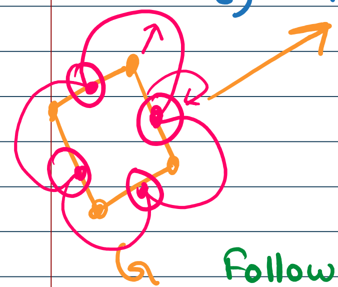
a) Fixing number (vertex)
b) Fixing number (edge)

} Defined fixing set

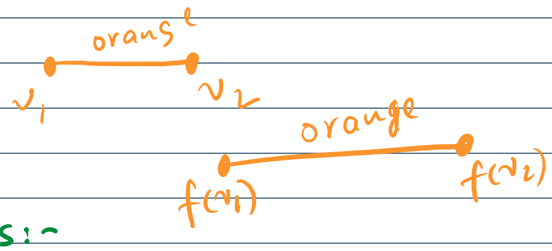


V.F.S. = $\{v_4, v_6\}$

$e(\ell(C_4)) = 4$

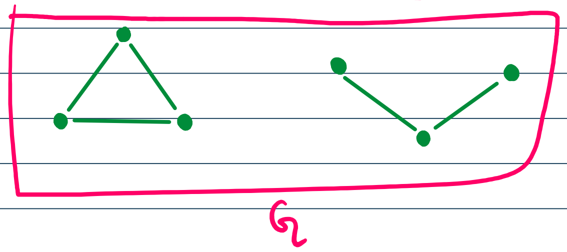


c) Asymm. Edge colouring number.



Follow up qs:-

1> (Assume everyone is familiar with line graphs)
Does there exist a relation between $a(G)$ and $a'(G)$?
or $f(G)$ or $f'(G)$?

2> 

Taking the Disjoint Union
The Automorphism grp of the graph $\cong$ Aut(C_6).

$\Rightarrow$ 2 graphs can have different $f(G)$'s but same $a(G)$s.

Hence, Linking $a(G)$, $f(G)$ might be possible (Group Theoritic Proof).

$f'(G)$ $f(G)$ $\boxed{a(G)}$ $a'(G)$

$f'(G)$ $f(G)$ $a'(G)$

Edge Vertex

Linear

37 Bounds ?